Exercise 80

Find h' in terms of f' and g'.

$$h(x) = \sqrt{\frac{f(x)}{g(x)}}$$

Solution

Calculate $h'(x)$ by using the chain and quotient rules.

$$\begin{aligned} h'(x) &= \frac{d}{dx}\sqrt{\frac{f(x)}{g(x)}} \\ &= \frac{1}{2}\left[\frac{f(x)}{g(x)}\right]^{-1/2} \cdot \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \\ &= \frac{1}{2}\sqrt{\frac{g(x)}{f(x)}} \cdot \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \\ &= \frac{1}{2\sqrt{f(x)}} \cdot \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^{3/2}} \\ &= \frac{f'(x)g(x) - g'(x)f(x)}{2\sqrt{f(x)[g(x)]^3}} \end{aligned}$$